RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, MAY 2025
THIRD YEAR [BATCH 2022-25]
MATHEMATICS [Honours]
Paper : DSE 3

Date : 14/05/2025
Time : 11 am – 1 pm
Full Marks : 50

[All notations have their usual meaning]

(Graph Theory & Number Theory)

Group - A

Answer all the questions. Maximum you can score is 30.

1. Use Dijkstra's shortest path algorithm to find the length of a shortest path from vertex z to all other vertices in the graph : [7]

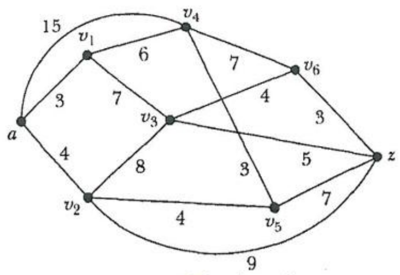

Fig1

2. Find the maximal flow and corresponding minimum cut for the s-t network : [8]

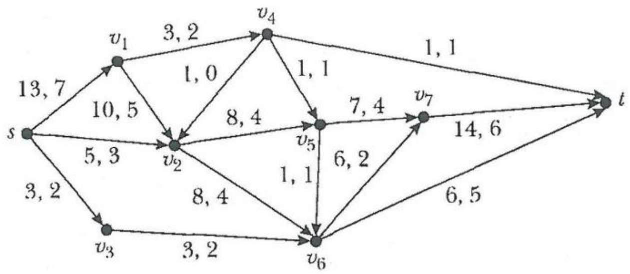

Fig 2

3. Give an example with proper justification of a graph which is Eulerian but not Hamiltonian. [3]

4. Use Kruskal's algorithm to find Minimum spanning tree of the graph in Fig 1(given in question no. 1). [5]

5. Prove that any connected graph G with n-vertices and (n-1) edges is a tree. [4]

6. For any connected plane graph with *n*-vertics, *e*-edges and *f*-faces, prove that $n-e+f=2$. [5]

7. Draw $K_{1,3}\square P_3$ and exhibit an optimal coloring of it. [2+2]

Group - B

Answer all the questions. Maximum you can score is 20.

8. Solve : $x^8 \equiv 10 \pmod{11}$ and $4^x \equiv 13 \pmod{17}$. [4+4]
9. Prove that for $k \geq 3$, the integer 2^k has no primitive root. [4]
10. Solve : $x^2 + 5x + 6 \equiv 0$ (mod 5^3). [4]
11. Prove that there are infinitely many primes of the form (8k-1). [4]
12. Evaluate : $(3658/12703)$ and $(1234/4567)$. [2+2]

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(Geometry of Curves and Surfaces)

Answer all the questions. Maximum you can score is 50.

1. Given a differentiable function $k(s), s \in (0,1)$, show that the parametrised curve having curvature $k(s)$ is given by $\gamma(s) = \left(\int_0^s \cos\theta(x)dx + a, \int_0^s \sin\theta(x)dx + b\right)$ where $\theta(x) = \int_0^x k(t)dt + \varphi$ and φ, a, b are constants. Further show that the curve is determined upto a translation of the vector *(a,b)* and a rotation of the angle φ . [15]

2. Let γ be a unit speed curve in $\mathbb{R}^3$ with constant curvature and zero torsion. Then show that γ is a parametrization of part of a circle. [8]

3. State and prove the iso-perimetric inequality for the plane curves. [10]

4. a) Show that the right circular cylinder $S = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1\}$ is a surface.

   b) If $T_r = \{(x,y,z) \in \mathbb{R}^2 \mid x^2 + y^2 = r^2\}, r \in \mathbb{R}^+$ is any other right circular cylinder, then T_r is diffeomorphic to *S*, given is part (a). [6+6]

5. Let S_1 and S_2 be smooth surfaces in $\mathbb{R}^3$. Define a local isometry $f : S_1 \to S_2$ between S_1 and S_2 . Prove that a local diffeomorphism $f : S_1 \to S_2$ is a local isometry if and only if for any surface patch σ_1 of S_1, the surface patches σ_1 of S_1 and $f \circ \sigma_1$ of S_2 have the same first fundamental form. [2+8]

6. Calculate the first fundamental form of the surface $\sigma(u,v) = (u - v, u + v, u^2 + v^2)$. [5]

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